AH-1540-S.E.-CV-19

M.A./M.Sc. (PREV.) MATHEMATICS

Term End Examination, 2019-20

REAL ANALYSIS AND MEASURE THEORY

PAPER-II

Time : Three Hours] **[Maximum Marks : 100**

Note : Attempt any five questions. All questions carry equal marks.

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1. (a) Prove that
$$\int_{-a}^{b} f dx \leq \int_{a}^{-b} f dx$$
(b) If $f \in R[a \ \ b]$ and x is monotonic increasing on $[a \ \ b]$ such that $x' \in R[a \ \ b]$, then prove $f \in R(x)$, and $\int_a^b f dx \leq \int_a^b f x' dx$
2. State and prove Riemann's theorem.
3. (a) Let x be monotonically increasing on $[a \ \ b]$ suppose $f_n \in R(x)$ on $[a \ \ b]$, for $n = 1, 2, 3, \dots\dots\dots$ and suppose $f_n \to F$ uniformly on $[a \ \ b]$. Then prove that $f \in R(x)$ on $[a \ \ b]$, and $\int_a^b f dx = \lim_{n\to\infty} \int_a^b f_n dx$
(b) Test for the uniform convergence of the series
$\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^5}{1+x^8} \dots\dots\dots , -1/_2 \leq x \leq \frac{1}{2}$
4. (a) If $\sum a_n, \sum b_n, \sum c_n$ converge to the sums A, B, C respectively, and if $c_n = a_0 b_n + a_1 b_{n-1} + \dots\dots + a_n b_0$, than prove that AB=C
(b) State and prove Tauber's theorem for power series.
5. (a) Prove that a linear operator A on a. Finite dimensional vector space X is one to one. If and only if the range of A is all of X.
(b) If Ω be the set of all invertible linear operator on R^n. Prove that if $A \epsilon \Omega, B \epsilon L(R^{n)}$ and $\| B - A \| . \| A^{-1} \| < 1$ then $B \epsilon \Omega$.
6. (a) State and prove chain rule.
(b) If X is complete metric space, and if. ϕ is a contraction of X into X, prove that there exists one and only one $x \epsilon X$ such that $\phi(x) = x$
7. (a) If E has the outer measure zero, then prove that E is measurable set. Also prove that every subset of E is measurable.
(b) Prove that if E_1 and E_2 are measurable sets, than so is $E_1 \cup E_2$
8. (a) Prove that a continuous function defined on a measurable set is measurable.
(b) Prove that if f and g are measurable functions, so is fog
9. (a) State and prove Bounded convergence theorem.
(b) State and prove Lebesgue Dominated convergence theorem.
10. (a) Let E be a measurable set with $m(E) < \infty$ prove that $L^\infty(E) \subset L^P(E)$ for each p with $1 \leq P < \infty$. Furthermore, if $f \epsilon L^\infty(E)$ then $\| f \|_\infty = \lim_{p\to\infty} \| f \|_p$
(b) Let $1 \leq p \leq \infty$ then, prove that for every pair f, $g \epsilon L^P$, the following inequality holds $\| f + g \|_p \leq \| f \|_p + \| g \|_p$